International Journal of Advances in Engineering and Management (IJAEM)
Volume 4, Issue 1 Jan 2022, pp: 166-170 www.ijaem.net ISSN: 2395-5252

Kushare Integral Transform for Newton's Law of Cooling

[1]Rohidas S.Sanap, [2]Dr.Dinkar P. Patil
[1]*Assistant Professor, Department of Mathematics and Statistics K.M.Agrawal College of Art's, Commerce and Science, Kalyan (W)*
[2]*Professor, Department of Mathematics,KR.T. Arts, B.H. Commerce and A.M. Science. College, Nashik*

Submitted: 01-01-2022 Revised: 05-01-2022 Accepted: 10-01-2022

ABSTRACT
The Newton's Law of Cooling occurs and plays important role in the field of physics. Also use of integral transform is easier technique in solving differential equations and boundary value problems. Lot of integral transforms are developed by many researchers. In this paper, we use the KUSHARE transform for solving Newton's Law of Cooling. We further solve the problems based on Newton's Law of Cooling by using newly introduced integral transform, KUSHARE transform.

Keywords: Newton's Law of Cooling, Ordinary differential equation,KUSHARE transform, Boundary value problems.

INTRODUCTION

Integral transform plays very important role in differential equations as well as integral equations. They are also very much useful in boundary value problems. Now a day lot of researchers have developed many integral transforms like Laplace, Sumudu, Elzaki, MahgoubMohand, Sawi, Shehu, Aboodh,Raj and many more. Recently in 2021 Kushare et al [1] introduced new integral transform named as KUSHARE transform. Soham transform [2] has been introduced in Nov. 2021 by Khakale et al. Many researchers have used various types of transforms for solving different types of differential equations, boundary value problems, Integral equations, System of differential equations and system of integral equations. Patil [3] used Sawi transform in Bessel functions. Laplace and Shehu transforms are used by him [4] in chemical sciences.Further he used Aboodh and Mahgoub transform for solving boundary value problems of system of ordinary differential equations [5]. Mahgoub transformis used for solving parabolic boundary value problems [6]. LoknathSahoo [7] used Laplace transform in Newton's Law of Cooling. Sawant [8] used Laplace transform in engineering field recently. Thus, there are variety of integral transforms developed by researchers and used it in number of fields.

Newton's Law of Cooling plays very much important role in physics and states that, "the rate of cooling of a body is directly proportional to the difference in temperatures of the body and the surroundings". If T is the temperature of the body at any time t then, Newton's law of coolingsatisfies an ordinary differential equation given by

$$\frac{dT}{dt} = -C(T - T_e) \qquad (1)$$

with initial condition at

$T(t_0) = T_0$

Where,T is the temperature of the object

T_e is the constant temperature of the environment(surroundings)

T_0 is the initial temperature of the object at time t_0

C is the constant of proportionality.

We shall organise this paper as follows. Preliminaries are stated in second section. In third section we apply KUSHARE transform to Newton's Law of Cooling. Fourth section is devoted to applications. Conclusion is stated in fifth section.

1. **Preliminaries:** In this section westate some preliminaries like definition of Kushare transform, transform of some functions, inverse transform, some properties and transform of derivatives, etc. which we need.

1.1. Definition of KUSHARE Transform:[1]

Let $A = \{f(t) \mid \exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\left(\frac{|t|}{\tau_j}\right)}, \text{if } t \in (-1)^j \times [0, \infty)\}$

For a given function in set A, the constant M must be finite number, τ_1, τ_2 may be finite or infinite, KUSHARE transform of function f is defined by operator

$$K[f(t)] = S(v) = v\int_0^{\infty} f(t)e^{-tv^{\alpha}}dt, t \geq 0, \tau_1 \leq v \leq \tau_2$$

where, α is any non-zero real number.

1.2. KUSHARE Transform of some functions

Sr. No.	f(t)	K[f(t)] = S(v)
1	1	$\frac{1}{v^{\alpha-1}}$
2	e^{-at}	$\frac{v}{v^{\alpha}+a}$

1.3. Linearity Property of KUSHARE Transform

If f(t) and g(t) are any two functions of t and α, β are any real constants such that K[f(t)] = S(v) and K[g(t)] = R(v), then $K[\alpha f(t) + \beta g(t)] = \alpha K[f(t)] + \beta K[g(t)] = \alpha S(v) + \beta R(v)$.

1.4. KUSHARE Transform of derivatives

If $K[f(t)] = S(v)$, then $K[f'(t)] = v^{\alpha}S(v) - vf(0)$

1.5. Inverse KUSHARE Transform

If K[f(t)] = S(v), then f(t) is called inverse KUSHARE transform of S(v) and denoted by $K^{-1}[S(v)]$

1.6. Inverse KUSHARE Transform of some functions

Sr. No.	S(v)	$K^{-1}[S(v)] = f(t)$
1	$\frac{1}{v^{\alpha-1}}$	1
2	$\frac{v}{v^{\alpha}+a}$	e^{-at}

2. APPLICATION OF KUSHARE TRANSFORM IN NEWTON'S LAW OF COOLING:

The Newton's Law of Cooling is given by equation (1)

$$\frac{dT}{dt} = -C(T - T_e)$$

Let $K[T(t)] = S(v)$

Taking KUSHARE transform on both sides, we get

$$K\left[\frac{dT}{dt}\right] = K[-C(T - T_e)]$$

$$\therefore v^{\alpha}S(v) - v\,T(0) = -CK(T) + C\,T_e\,K(1)$$

$$\therefore v^{\alpha}S(v) - vT_0 = -CS(v) + CT_e\left(\frac{1}{v^{\alpha-1}}\right)$$

$$\therefore v^{\alpha}S(v) + CS(v) = CT_e\left(\frac{1}{v^{\alpha-1}}\right) + vT_0$$

$$\therefore (v^{\alpha} + C)S(v) = CT_e\left(\frac{1}{v^{\alpha-1}}\right) + vT_0$$

$$\therefore S(v) = T_e\left\{\frac{C}{v^{\alpha-1}(v^{\alpha}+C)}\right\} + T_0\left\{\frac{v}{v^{\alpha}+C}\right\}$$

$$\therefore S(v) = vT_e\left\{\frac{C}{v^{\alpha}(v^{\alpha}+C)}\right\} + T_0\left\{\frac{v}{v^{\alpha}+C}\right\}$$

We use partial fraction and obtain,

$$\frac{C}{v^{\alpha}(v^{\alpha}+C)} = \frac{1}{v^{\alpha}} - \frac{1}{v^{\alpha}+C}$$

$$\therefore S(v) = vT_e\left\{\frac{1}{v^{\alpha}} - \frac{1}{v^{\alpha}+C}\right\} + T_0\left\{\frac{v}{v^{\alpha}+C}\right\}$$

$$\therefore S(v) = T_e\left\{\frac{1}{v^{\alpha-1}}\right\} + (T_0 - T_e)\left\{\frac{v}{v^{\alpha}+C}\right\}$$

Let $K^{-1}[S(v)] = T(t)$

Now, taking Inverse KUSHARE transform on both sides, we get

$$K^{-1}[S(v)] = K^{-1}\left[T_e\left\{\frac{1}{v^{\alpha-1}}\right\} + (T_0 - T_e)\left\{\frac{v}{v^{\alpha}+C}\right\}\right]$$

$$\therefore T(t) = T_e K^{-1}\left\{\frac{1}{v^{\alpha-1}}\right\} + (T_0 - T_e)K^{-1}\left\{\frac{v}{v^{\alpha} + C}\right\}$$
$$\therefore T(t) = T_e + (T_0 - T_e)e^{-Ct}$$

3. APPLICATIONS OF KUSHARE TRANSFORM:

In this section we solve some problems based on Newton's Law of Cooling by using Kushare integral transform

Problem (1)

A hot coffee with initial temperature of 115°F is kept in a room temperature of 35°F. The rate of change of temperature is 20°F per / min. how long it will take coffee to cool to a temperature of 40°F ?

Solution: Assuming that a coffee obeys Newton's Law of Cooling, we have

$$\frac{dT}{dt} = -C(T-35), T(0) = 115, T'(0) = -20$$

First, we will find the value of C by using initial condition

-20 = -C (115 – 35)

-20 = -80C

$$C = 0.25$$

So, the differential equation can be written as

$$\frac{dT}{dt} = -0.25(T - 35)$$

Now, by KUSHARE transform, we get

$$\therefore v^{\alpha} S(v) - v\,T(0) = -0.25K(T) + 0.25 \times 35\,K(1)$$
$$\therefore v^{\alpha} S(v) - 115v = -0.25S(v) + 0.25 \times 35\left\{\frac{1}{v^{\alpha-1}}\right\}$$
$$\therefore (v^{\alpha} + 0.25)S(v) = 0.25 \times 35\left\{\frac{1}{v^{\alpha-1}}\right\} + 115v$$
$$\therefore S(v) = 35\left\{\frac{0.25}{v^{\alpha-1}(v^{\alpha} + 0.25)}\right\} + 115\left\{\frac{v}{v^{\alpha} + 0.25}\right\}$$
$$\therefore S(v) = 35v\left\{\frac{0.25}{v^{\alpha}(v^{\alpha} + 0.25)}\right\} + 115\left\{\frac{v}{v^{\alpha} + 0.25}\right\}$$
$$\therefore S(v) = 35v\left\{\frac{1}{v^{\alpha}} - \frac{1}{v^{\alpha} + 0.25}\right\} + 115\left\{\frac{v}{v^{\alpha} + 0.25}\right\}$$
$$\therefore S(v) = 35\left\{\frac{1}{v^{\alpha-1}}\right\} + (115 - 35)\left\{\frac{v}{v^{\alpha} + 0.25}\right\}$$
$$\therefore S(v) = 35\left\{\frac{1}{v^{\alpha-1}}\right\} + 80\left\{\frac{v}{v^{\alpha} + 0.25}\right\}$$

By taking inverse KUSHARE Transform, we get

$$T(t) = 35 + 80e^{-0.25t}$$

Putting, T = 40 in above, we get

$$40 = 35 + 80e^{-0.25t}$$
$$e^{-0.25t} = \frac{1}{16}$$
$$e^{0.25t} = 16$$

0.25t = ln 16

0.25t = 2.7725887222

t = 11.0903548888 min

Coffee will take 11.09 minutes for cooling to a temperature of 40°F.

Problem (2)

A heated metal beam cools at the rate of 3°C per minute when its temperature is 50°C. Find the time taken to cool at 36°C if the temperature of the surroundings is 27°C

Solution: Assuming that a heated metal beam obeys Newton's Law of Cooling, we have

$$\frac{dT}{dt} = -C(T-27), T(0) = 50, T'(0) = -3$$

First, we will find the value of C by using initial condition

-3 = -C (50 – 27)

-3 = -23C

C = 0.13

The differential equation can be written as

$$\frac{dT}{dt} = -0.13(T - 27)$$

Now, by KUSHARE transform, we get

$$\therefore\ v^{\alpha}S(v) - v\,T(0) = -0.13K(T) + 0.13 \times 27\ K(1)$$

$$\therefore\ v^{\alpha}S(v) - 50v = -0.13S(v) + 0.13 \times 27\left\{\frac{1}{v^{\alpha-1}}\right\}$$

$$\therefore\ (v^{\alpha} + 0.13)S(v) = 0.13 \times 27\left\{\frac{1}{v^{\alpha-1}}\right\} + 50v$$

$$\therefore\ S(v) = 27\left\{\frac{0.13}{v^{\alpha-1}(v^{\alpha} + 0.13)}\right\} + 50\left\{\frac{v}{v^{\alpha} + 0.13}\right\}$$

$$\therefore\ S(v) = 27v\left\{\frac{0.13}{v^{\alpha}(v^{\alpha} + 0.13)}\right\} + 50\left\{\frac{v}{v^{\alpha} + 0.13}\right\}$$

$$\therefore\ S(v) = 27v\left\{\frac{1}{v^{\alpha}} - \frac{1}{v^{\alpha} + 0.13}\right\} + 50\left\{\frac{v}{v^{\alpha} + 0.13}\right\}$$

$$\therefore\ S(v) = 27\left\{\frac{1}{v^{\alpha-1}}\right\} + (50 - 27)\left\{\frac{v}{v^{\alpha} + 0.13}\right\}$$

$$\therefore\ S(v) = 27\left\{\frac{1}{v^{\alpha-1}}\right\} + 23\left\{\frac{v}{v^{\alpha} + 0.13}\right\}$$

By taking inverse KUSHARE Transform, we get

$$T(t) = 27 + 23e^{-0.13t}$$

Putting, T = 36 in above, we get

$$36 = 27 + 23e^{-0.13t}$$

$$e^{-0.13t} = \frac{9}{23}$$

$$e^{0.13t} = \frac{23}{9}$$

0.13t = ln2.56
0.13t = 0.94
t = 7.23 min
A heated metal beam will take 7.23 minutes for cooling to a temperature of 36℃.

CONCLUSION

In this paper, we have successfully used KUSHARE Integral Transform for solving Newton's Law of Cooling. We get the solutions of the problems based on Newton's Law of Cooling easily and accurate.

ACKNOWLEDGEMENT

We are thankful to theDepartment of Mathematics, KRT Arts, BH Commerce and AM Science College, Nashik and Department of Mathematics and Statistics, K. M. Agrawal College of Art's, Commerce and Science, Kalyan (W) for providing me the facility of well-equipped library and ICT facility with internet.

REFERENCES

[1]. Sachin Ramdas Kushare, D.P. Patil, ArchanaMadhukarTakate, The New Integral Transform "KUSHARE" Transform, International Journal of Advances in Engineering and Management (IJAEM) Volume 3, Issue 9 (2021) pp. 1589-1592.

[2]. Savita Khakale, D. P. Patil,The New Integral Transform, "Soham transform" International Journal of Advances in Engineering and Management (IJAEM) Volume 3, Issue 10 (2021) pp. 126-132.

[3]. D. P. Patil, Application of Sawi Transform in Bessel Function, Aayushi International Interdisciplinary Research Journal, Issue 86 (2021), pp. 171-175.

[4]. D. P. Patil, Application of Integral Transform (Laplace and Shehu) in Chemical Sciences, Aayushi International Interdisciplinary Research Journal, Issue 88 (2021), pp. 437-441.

[5]. D. P. Patil, Aboodh and Mahgoub Transform in Boundary Value Problem of System of Ordinary Differential Equations, International Journal of Advanced Research in Science, Communication and Technology Vol. 6, Issue 11 (2021) pp. 67-75.

[6]. D. P. Patil, Applications of Mahgoub Transform in Parabolic Boundary Value Problems, International Journal of Current

Advanced Research, Vol. 9, Issue 4 (2020) pp. 21949-21951.

[7]. LokanathSahoo, "Application of Laplace Transform for solving problems on Newton's Law of Cooling", International Journal of Recent Scientific Research Vol. 11, Issue, 12(A), (2020) pp. 40207-40209.

[8]. L. S. Sawant, Applications of Laplace Transform in Engineering Fields, International Research Journal of Engineering and Technology, Vol. 5, Issue 5 (2018) pp. 3100 -3105